VISCOUS DAMPING ESTIMATION OF STRUCTURES USING EXTREME FREQUENCY BANDWIDTH METHOD

XÁC ĐỊNH CẢN NHỚT CỦA KẾT CẤU SỬ DỤNG PHƯƠNG PHÁP DẢI TẦN SỐ CỰC TRỊ

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Abstract: *The classical half-power bandwidth (HPB) method is known as a simple and widely used method to identify damping in experimental research on structural vibrations. However, this method is only effective in systems with small damping and separate natural frequencies. This paper presents the extreme frequency bandwidth (EFB) method and proposes a formula to estimate the viscous damping ratio from the extreme points of the real part of the frequency response function (FRF) in the displacement spectrum analysis of structures. The displacement FRF is obtained by Fourier transform of the simulated load and response signals of the system. The results show that the EFB method can identify the viscous damping in structures that have one or more degrees of freedom with different damping levels.*

Keywords: Viscous damping estimation, extreme frequency bandwidth method, half-power bandwidth method, structural vibration, frequency response function.

Tóm tắt: *Phương pháp dải tần số nửa công suất là phương pháp đơn giản và được sử dụng rộng rãi để nhận dạng cản trong nghiên cứu thực nghiệm về dao động kết cấu. Tuy nhiên, phương pháp này chỉ hiệu quả trong hệ có cản nhỏ và các tần số riêng tách biệt. Bài báo này trình bày phương pháp dải tần số cực trị và đề xuất một công thức để ước lượng tỷ số cản nhớt từ các điểm cực trị của đồ thị phần thực trong phân tích phổ chuyển vị của kết cấu. FRF chuyển vị thu được bằng biến đổi Fourier các tín hiệu mô phỏng tải trọng và phản ứng của hệ. Kết quả cho thấy phương pháp dải tần số cực trị có thể nhận dạng cản nhớt trong kết cấu có một hoặc nhiều bậc tự do với các mức cản khác nhau.*

Từ khóa: *Ước lượng cản nhớt, phương pháp dải tần số cực trị, phương pháp dải tần số nửa công suất, dao động kết cấu, hàm phản ứng tần số.*

1. Introduction

Damping parameters play a crucial role in structural vibration analysis. These parameters are not easily determined through theoretical means and are often estimated experimentally. Damping estimation methods can be classified into several categories, including estimation for individual modes, estimation for multiple modes [1-3], time-domain estimation [4, 5], and frequency-domain estimation [6, 7] and operational modal analysis [8].

The classical half-power bandwidth (HPB) method is a popular technique for determining damping from experimental structural vibration data. This approach calculates the viscous damping ratio using a classical formula based on experimental measurements of the displacement or acceleration frequency response function (FRF). This ratio is equal to half the frequency bandwidth which the FRF signal power is reduced to half of its maximum value [3].

The classical formula for determining damping is an approximate method that is typically used when the damping ratio of a structure is very small. However, when damping is high, this formula can yield significant errors. To improve the accuracy, more advanced approximate formulas based on the HPB method have been developed. Yin [9, 10] proposed a more precise formula for calculating the damping ratio of a single degree-of-freedom (DOF) system using the squared FRF curve. Olmos (2010) [11] identified damping in multi-DOF systems using the HPB approach, assuming that viscous damping is proportional to stiffness and mass and remains constant.

Wang I [12] proposed a third-order formula based on the HPB method for estimating the viscous damping ratio in single-degree-of-freedom (DOF) systems. Papagiannopoulos et al. [13] developed the third-order formulas to estimate damping ratios in multi-DOF systems. Then, Wang J [14, 15] compared the third-order formula to the classical formula for determining the damping ratios in 2-DOF systems. Generally, the larger the separation between natural frequencies of the structure, the smaller the errors in damping estimation, and the accuracy of the HPB method improves as the damping level decreases. In 2014, Wu [16] proposed a formula for determining the damping ratio by neglecting the sixth-order infinitesimal term, which provides high accuracy for estimating the damping ratio in single-DOF systems. However, a limitation of Wu's formula, as well as a general limitation of HPB methods, is that they are mainly applicable within a damping range of 0 to 0.383. Additionally, these damping estimation formulas are designed for single-DOF systems and require specific adjustments when applied to multi-DOF systems.

Vu Dinh Huong et al. [17] employed the general bandwidth method to develop a precise formula for predicting the viscous damping ratio based on the displacement spectrum of a single-DOF system. This formula enhances the determination of the damping ratio by using a power factor instead of the factor of 2 used in the HPB method. Analysis of the single-DOF system showed that the method provides a more accurate damping estimate than the HPB method and is effective for estimating high levels of damping. Subsequently, the authors applied the general bandwidth method to identify hysteresis damping from acceleration frequency response functions (FRF) [18, 20]. Later, Wu [19] also analyzed the displacement spectrum and proposed a formula for estimating the damping ratio based on the power ratio, similar to the formula in [17]. The authors then examined how the power ratio influences the accuracy of damping estimation formulas in multi-DOF systems.

The EFB method is a recently developed approach for estimating damping. It relies on the real part of the frequency response function (FRF) to create formulas for estimating damping parameters. This paper introduces an approximate formula for estimating the viscous damping ratio using the EFB method. Numerical experiments were conducted on single-degree-of-freedom (DOF) and multi-DOF structures, and the results of viscous damping estimation using the proposed formula are compared with existing formulas of the half-power method.

2. Half-power bandwidth method

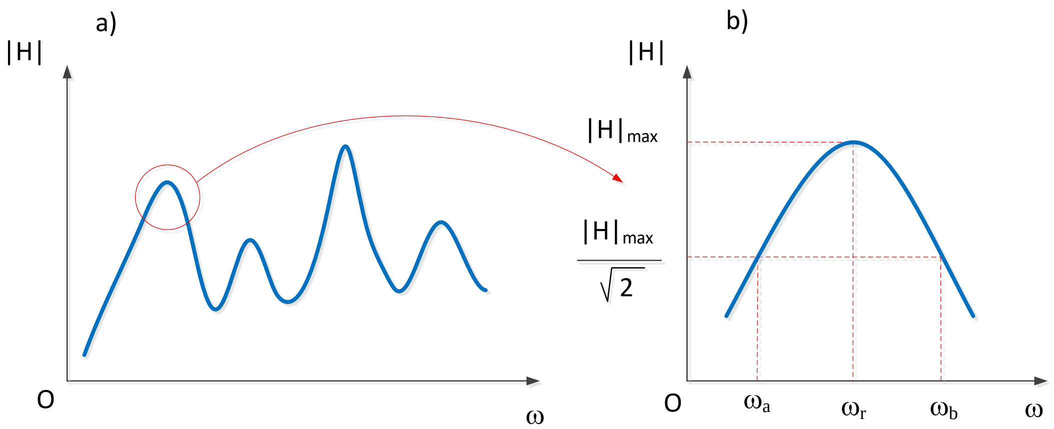

Figure 1. *Half-power bandwidth method*

The classical formula for damping estimation based on the displacement FRF of single-DOF system has the following form [1-3]:

$$\xi = \frac{\omega_b - \omega_a}{2\omega_r} = \frac{b}{2} \quad (1)$$

where b is the frequency bandwidth; ω_a and ω_b are two frequencies at the amplitudes equal to $1/\sqrt{2}$ the maximum amplitude (at resonance frequency ω_r).

By continuously expanding the square root expressions, in 2011, Wang [6] proposed a third-order equation to determine the viscous damping ratio from the displacement FRF as follows:

$$4\xi^3 + 2\xi = b \quad (2)$$

The formula for determining ξ according to (2) has significantly reduced the error, but the condition for applying (2) has been shown by Papagiannopoulos [2] and Wu [5] to be $\xi < 0.383$.

Furthermore, the error when estimating the larger damping ratio ($\xi \geq 0.35$) is still greater than 10%.

3. Extreme frequency bandwidth method

Dynamic equilibrium equation of a single-DOF system with viscous damping:

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=p(t) \quad (3)$$

Transform to the frequency domain, substituting equations u(t) = U(ω).e$^{i\omega t}$ and p(t) = P(ω).e$^{i\omega t}$ into equation (3), we obtain:

$$(-m\omega^2+ic\omega+k).U(\omega).e^{i\omega t}=P(\omega).e^{i\omega t} \quad (4)$$

The ratio between displacement and applied force in the frequency domain:

$$H(\omega)=\frac{U(\omega)}{P(\omega)}=\frac{1}{k-m\omega^2+ic\omega} \quad (5)$$

H(ω) is called the displacement (or receptance) frequency response function (FRF) of the single-DOF system. FRF is a complex quantity that depends on frequency and it can also be expressed as.

$$H(\omega)=\frac{1/k}{1-\dfrac{\omega^2}{\omega_0^2}+i2\xi\dfrac{\omega}{\omega_0}}=\frac{1}{k}\frac{1-\eta^2}{(1-\eta^2)^2+(2\xi\eta)^2}-i\frac{1}{k}\frac{2\xi\eta}{(1-\eta^2)^2+(2\xi\eta)^2} \quad (6)$$

with, $\eta=\omega/\omega_0$

Real part of the receptance FRF is:

$$\text{Re(H)}=\frac{1}{k}\frac{1-\eta^2}{(1-\eta^2)^2+(2\xi\eta)^2} \quad (7)$$

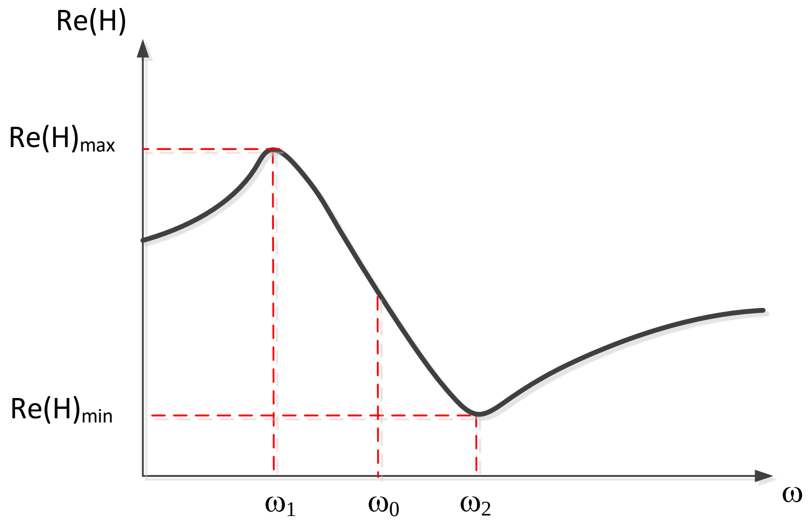

Figure 2. *Extreme frequency bandwidth method*

The graph of the Re(H) function is shown in Figure 2, where ω_1 and ω_2 are the extreme frequencies. The function Re(H) reaches an extreme value when its derivative is zero:

$$\frac{d(\text{Re(H)})}{d\eta}=0\Rightarrow\eta^4-2\eta^2+1-4\xi^2=0 \quad (8)$$

The solution of the equation (8) will then give the following two roots:

$$\eta_{1,2}=\sqrt{1\mp2\xi}\ \text{(with } \xi\leq0.5) \quad (9)$$

Using Taylor expansion for the expressions (9), we have:

$$\frac{\omega_1}{\omega_0}=\eta_1=\sqrt{1-2\xi}=1-\frac{1}{2}(2\xi)-\frac{1}{2}\xi^2-\frac{1}{2}\xi^3-\frac{5}{8}\xi^4+\cdots \quad (10)$$

$$\frac{\omega_2}{\omega_0}=\eta_2=\sqrt{1+2\xi}=1+\frac{1}{2}(2\xi)-\frac{1}{2}\xi^2+\frac{1}{2}\xi^3-\frac{5}{8}\xi^4+\cdots \quad (11)$$

From (10) and (11), obtain the formula to determine the natural frequency of the structure according to the extreme frequencies:

$$\frac{\omega_2+\omega_1}{\omega_0}=\eta_2+\eta_1=2-\xi^2+\frac{5}{4}\xi^4+... \quad (12)$$

When the damping is small (ξ << 1), ignoring the 2nd-order infinitesimal quantity of ξ and its higher-order terms in (12), we get the natural frequency approximately equal to the average of the extreme frequencies:

$$\omega_0 \approx \frac{\omega_2+\omega_1}{2} \quad (13)$$

Combining (10) and (11) yields the extreme frequency bandwidth and ignoring the 5th-order infinitesimal quantity of ξ we get a third-order equation to determine the damping ratio:

$$\beta=\eta_2-\eta_1=2\xi+\xi^3 \quad (14)$$

In which, β is extreme frequency bandwidth of real part of receptance FRF:

$$\beta=\frac{\omega_2-\omega_1}{\omega_0} \quad (15)$$

Finally, the cubic equation (14) has a solution:

$$\xi=\left(\frac{\beta}{2}+\sqrt{\frac{\beta^2}{4}+\frac{8}{27}}\right)^{1/3}-\frac{2}{3}\left(\frac{\beta}{2}+\sqrt{\frac{\beta^2}{4}+\frac{8}{27}}\right)^{-1/3} \quad (16)$$

Equation (16) is an approximate formula, proposed to determine the damping ratio from extreme frequency bandwidth of real part of displacement FRF. The estimated range of the damping ratio of this method according to (9) is ξ ≤ 0.5.

4. Comparison of the accuracy of damping estimation methods in the single-DOF system

Consider a single-DOF system with natural frequency ω_0 = π (rad/s). First, assume the damping ratio ξ is a value from 0.05 to 0.5, then equation (6) gives the dimensionless FRF (the stiffness *k* can be assumed to be 1). From this, the amplitude of the FRF and the corresponding half-power frequencies can be determined, and similarly the real part of the FRF and the corresponding extreme frequencies can be obtained. Finally, the damping ratio can be estimated according to the above formulas.

The results of the damping estimation by the methods are presented in Table 1. In which, the first column is the assumed ξ value, the second and third columns show the ξ value calculated by the classical HPB method using the classical formula (1) and its corresponding error (in %); the fourth and fifth columns are the ξ values calculated by the third-order formula (2) and the error; the sixth and seventh columns show the damping ratio value and the error estimated by the proposed formula from the extreme frequency bandwidth method in this paper.

Table 1. *Estimation of damping ratio in the single-DOF system*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.050	0.5%	0.050	0.01%	0.050	0.12%
0.1	0.102	2.1%	0.100	0.06%	0.101	0.51%
0.15	0.157	4.8%	0.150	0.29%	0.152	1.19%
0.2	0.218	9.1%	0.202	0.91%	0.204	2.22%
0.25	0.289	15.6%	0.256	2.23%	0.259	3.69%
0.3	0.377	25.6%	0.315	4.84%	0.317	5.79%
0.35	0.502	43.5%	0.387	10.46%	0.381	8.79%
0.383	0.707	84.7%	0.483	26.00%	0.427	11.56%
0.4	-	-	-	-	0.453	13.35%
0.45	-	-	-	-	0.546	21.24%
0.5	-	-	-	-	0.771	54.18%

Figure 3 shows more clearly the error of the damping estimation methods when the damping ratio changes. Accordingly, the classical formula (1) gives the largest error and is only suitable for structures with a damping ratio less than 0.15. The formula (2) proposed by Wang [4] can estimate well for a damping ratio up to 0.3. However, when the damping ratio is larger than the critical value $\xi_{th} \approx 0.383$, the both formulas give large errors and are unstable. The proposed formula estimates the damping quite accurately, similar to Wang's third-order formula when the structure has damping level less than 0.3. The proposed method can estimate the damping ratio up to 0.5, while HPB method can only estimate the damping ratio up to 0.383.

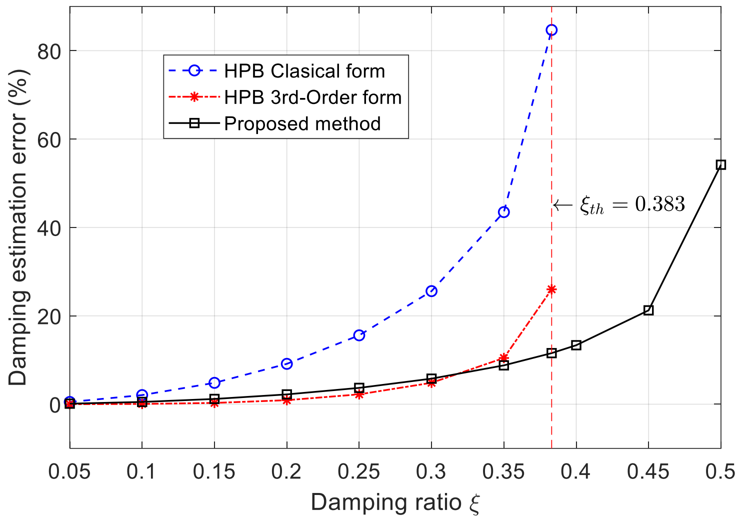

Figure 3. *The error of the damping estimation methods*

5. Numerical simulation on structures

5.1 Simulated vibration measurement data

Consider the structure of a 3-story reinforced concrete building as shown in Figure 4, which is subjected to dynamic load. The structure has the following parameters: beam cross-section size bxh = 0.22 x 0.5 cm, column cross-section 0.22 x 0.22 cm, floor height is 5 m, span width is 6 m. Concrete material has modulus of elasticity E = 2.5e7 (kN/m^2), specific weight ρ = 25 (kN/m^3).

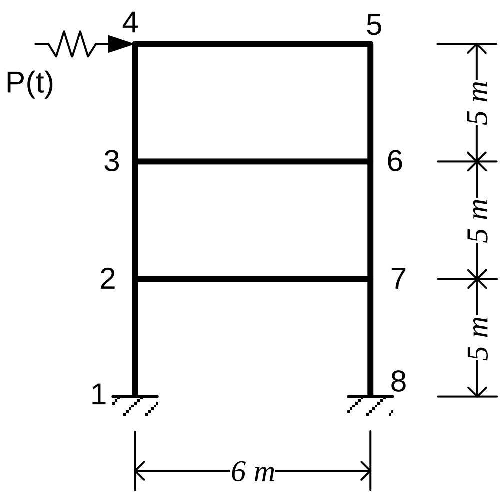

Figure 4. *The 3-story building*

The system is subjected to a half cycle of a sinusoidal load with period 2.0 s as shown in Figure 5a. Assume that the proportional damping model has the same damping levels for all modes. The displacement simulation measurement data at the nodes is obtained by time history analysis of the finite element model using the direct integration method. The horizontal displacement at node 5 with small damping level (ξ = 0.05÷0.15) is shown in figure 5b and with large damping level (ξ = 0.2÷0.4) it is shown in Figure 6b.

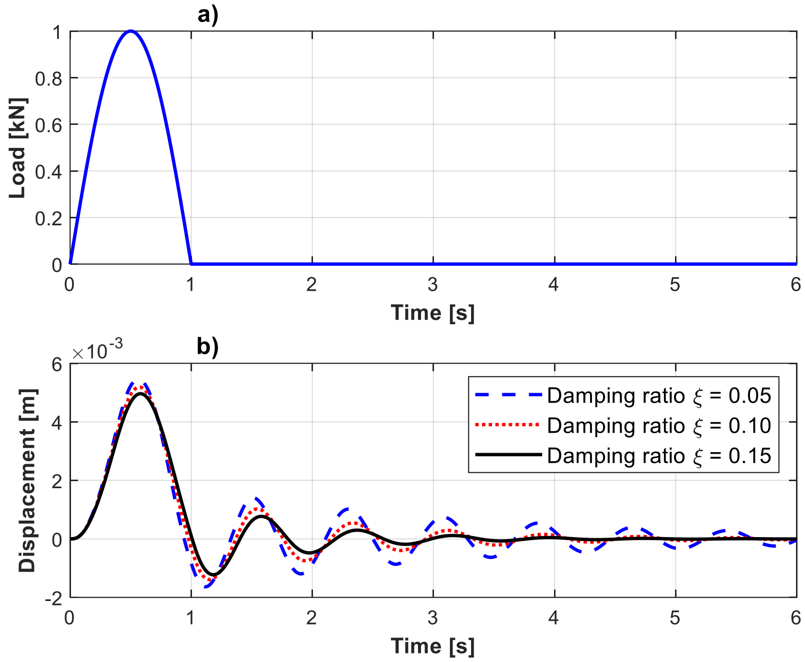

Figure 5. *The load and displacement at node 5 with small damping level*

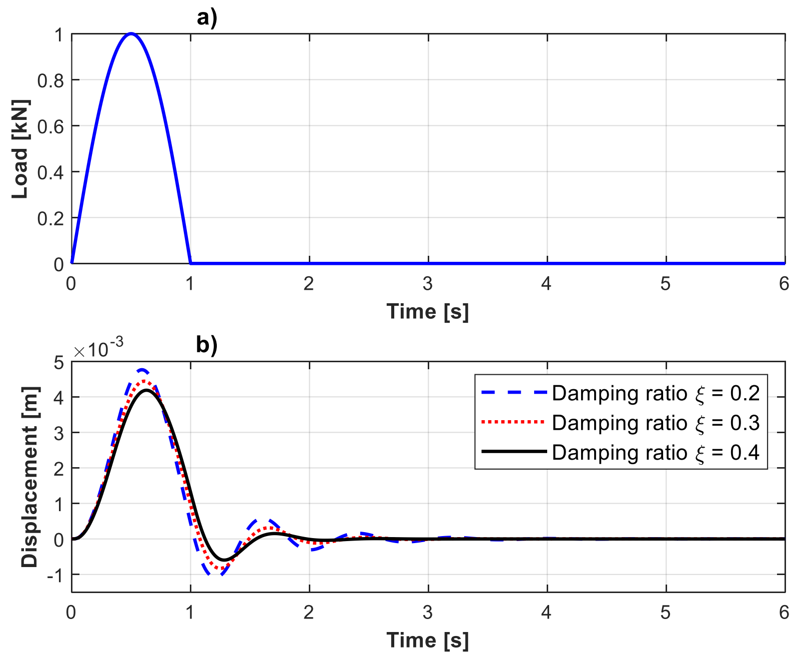

Figure 6. *The load and displacement at node 5 with large damping level*

5.2 The result of damping estimation

Simulate the displacement frequency response function using Fourier transform of the input-output responses as follows:

$$H(\omega)=\frac{U(\omega)}{P(\omega)} \tag{17}$$

where U(ω) and P(ω) are the Fourier transforms of the displacement u(t) and the dynamic load p(t) respectively.

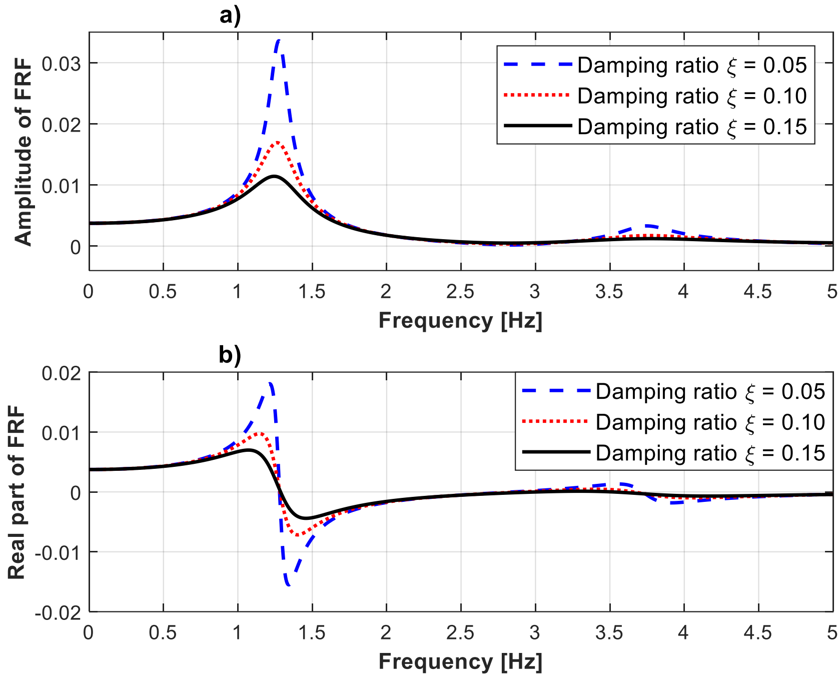

Figure 7. *The FRF with small damping*

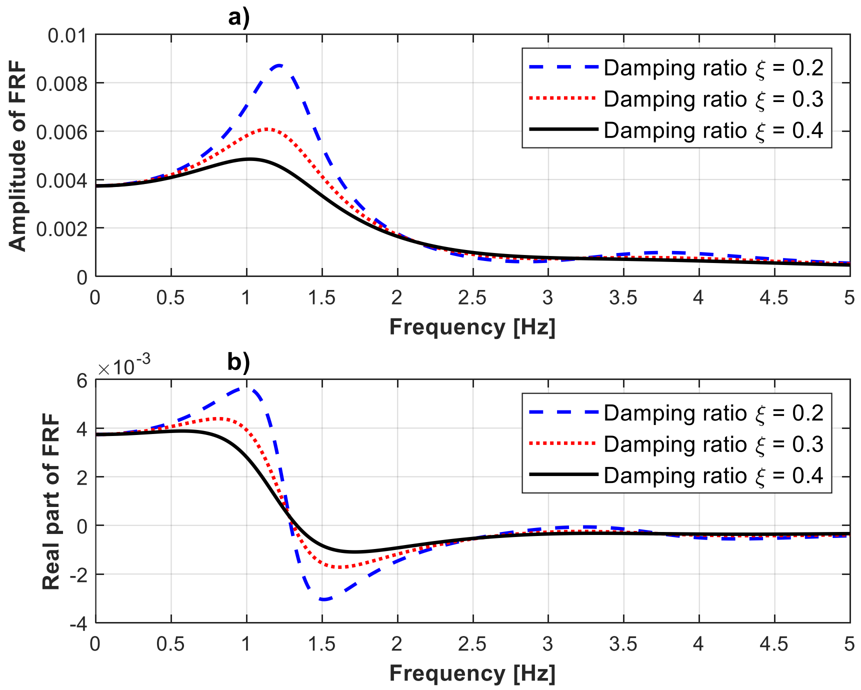

Figure 8. *The FRF with large damping*

The graphs of amplitude and real part of the FRF for horizontal displacement at node 5 are shown in Figure 7 in case the assumed damping level is small. The corresponding graphs in case of large damping level are shown in Figure 8.

From the graphs, the half-power frequencies in amplitude of the FRF and the extreme frequencies in real part of FRF can be determined. From there, the damping ratios can be estimated using the half-power bandwidth method and extreme frequency bandwidth method. Finally, calculate the damping ratios according to the above formulas and evaluate its error. The damping estimation results for the first mode are provided in Table 2, and those for the second mode are in Table 3.

Table 2. *Damping estimation error for the first mode*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.0502	0.4%	0.0499	0.1%	0.0500	0.03%
0.1	0.1025	2.5%	0.1005	0.5%	0.1000	0.05%
0.15	0.1596	6.4%	0.1525	1.7%	0.1513	0.86%
0.2	0.225	12.5%	0.207	3.6%	0.203	1.7%
0.3	0.409	36.3%	0.334	11.4%	0.316	5.2%
0.4	-	-	-	-	0.452	13.0%

Table 2 show that the damping estimation error for the first mode using the proposed method is significantly smaller compared to the classical and 3-rd order formulas based on HPB method. In addition, the proposed method provides a larger damping estimation range than the existing formulations. For the first mode, the classical formula gives good results when estimating small damping levels (about less than 0.1), the 3rd-order formula can estimate larger damping levels (about 0.2), and the proposed formula can estimate damping ratios up to 0.3 with an error of about 5%. Additionally, at a high damping level (0.4), the proposed method has an error of 13%, while the HPB method does not provide a solution.

Table 3. *Damping estimation error of the second mode*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.049	1.9%	0.049	2.4%	0.049	2.1%
0.1	0.096	3.8%	0.094	5.5%	0.095	5.3%
0.15	0.141	6.0%	0.136	9.3%	0.135	9.8%
0.2	0.183	8.6%	0.172	13.8%	0.171	14.7%
0.3	0.338	15.4%	0.248	20.8%	0.234	22.1%
0.4	-	-	-	-	0.284	29.1%

Table 3 shows that all three formulas give good results only when estimating small damping levels (less than 0.1). As the damping level increases, the errors of the formulas also increase. The proposed formula has lower accuracy than the 3rd-order formula, but the difference is quite small. On the other hand, at high damping levels, the classical formula for damping estimation performs better for the second mode compared to the other two formulas, although the error is still relatively large (from 6% to 15.4%). The proposed method can still estimate a damping level of 0.4, but with an error as high as 29.1%.

Tables 2 and 3 also show that the proposed method is highly effective for estimating damping in the first mode, even at high damping levels. However, the method performs less effectively when applied to the second mode, especially at high damping levels.

5. Conclusion

The paper proposes a formula for estimating the viscous damping ratio of a structure from the extreme frequency bandwidth of the real part graph of the displacement FRF. For a single-DOF system, the proposed formula gives more accurate damping estimation results than the classical formula, and the damping estimation range is extended to a damping level of 0.5. In a multi-DOF system, the proposed formula can estimate the damping ratio for the first mode with small errors, even with a large damping level. However, the damping ratio estimation results for the second mode increase significantly as the damping level increases.

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VISCOUS DAMPING ESTIMATION OF STRUCTURES USING EXTREME FREQUENCY BANDWIDTH METHOD

XÁC ĐỊNH CẢN NHỚT CỦA KẾT CẤU SỬ DỤNG PHƯƠNG PHÁP DẢI TẦN SỐ CỰC TRỊ

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Abstract: *The classical half-power bandwidth (HPB) method is known as a simple and widely used method to identify damping in experimental research on structural vibrations. However, this method is only effective in systems with small damping and separate natural frequencies. This paper presents the extreme frequency bandwidth (EFB) method and proposes a formula to estimate the viscous damping ratio from the extreme points of the real part of the frequency response function (FRF) in the displacement spectrum analysis of structures. The displacement FRF is obtained by Fourier transform of the simulated load and response signals of the system. The results show that the EFB method can identify the viscous damping in structures that have one or more degrees of freedom with different damping levels.*

Keywords: Viscous damping estimation, extreme frequency bandwidth method, half-power bandwidth method, structural vibration, frequency response function.

Tóm tắt: *Phương pháp dải tần số nửa công suất là phương pháp đơn giản và được sử dụng rộng rãi để nhận dạng cản trong nghiên cứu thực nghiệm về dao động kết cấu. Tuy nhiên, phương pháp này chỉ hiệu quả trong hệ có cản nhỏ và các tần số riêng tách biệt. Bài báo này trình bày phương pháp dải tần số cực trị và đề xuất một công thức để ước lượng tỷ số cản nhớt từ các điểm cực trị của đồ thị phần thực trong phân tích phổ chuyển vị của kết cấu. FRF chuyển vị thu được bằng biến đổi Fourier các tín hiệu mô phỏng tải trọng và phản ứng của hệ. Kết quả cho thấy phương pháp dải tần số cực trị có thể nhận dạng cản nhớt trong kết cấu có một hoặc nhiều bậc tự do với các mức cản khác nhau.*

Từ khóa: *Ước lượng cản nhớt, phương pháp dải tần số cực trị, phương pháp dải tần số nửa công suất, dao động kết cấu, hàm phản ứng tần số.*

1. Introduction

Damping parameters play a crucial role in structural vibration analysis. These parameters are not easily determined through theoretical means and are often estimated experimentally. Damping estimation methods can be classified into several categories, including estimation for individual modes, estimation for multiple modes [1-3], time-domain estimation [4, 5], and frequency-domain estimation [6, 7] and operational modal analysis [8].

The classical half-power bandwidth (HPB) method is a popular technique for determining damping from experimental structural vibration data. This approach calculates the viscous damping ratio using a classical formula based on experimental measurements of the displacement or acceleration frequency response function (FRF). This ratio is equal to half the frequency bandwidth which the FRF signal power is reduced to half of its maximum value [3].

The classical formula for determining damping is an approximate method that is typically used when the damping ratio of a structure is very small. However, when damping is high, this formula can yield significant errors. To improve the accuracy, more advanced approximate formulas based on the HPB method have been developed. Yin [9, 10] proposed a more precise formula for calculating the damping ratio of a single degree-of-freedom (DOF) system using the squared FRF curve. Olmos (2010) [11] identified damping in multi-DOF systems using the HPB approach, assuming that viscous damping is proportional to stiffness and mass and remains constant.

Wang I [12] proposed a third-order formula based on the HPB method for estimating the viscous damping ratio in single-degree-of-freedom (DOF) systems. Papagiannopoulos et al. [13] developed the third-order formulas to estimate damping ratios in multi-DOF systems. Then, Wang J [14, 15] compared the third-order formula to the classical formula for determining the damping ratios in 2-DOF systems. Generally, the larger the separation between natural frequencies of the structure, the smaller the errors in damping estimation, and the accuracy of the HPB method improves as the damping level decreases. In 2014, Wu [16] proposed a formula for determining the damping ratio by neglecting the sixth-order infinitesimal term, which provides high accuracy for estimating the damping ratio in single-DOF systems. However, a limitation of Wu's formula, as well as a general limitation of HPB methods, is that they are mainly applicable within a damping range of 0 to 0.383. Additionally, these damping estimation formulas are designed for single-DOF systems and require specific adjustments when applied to multi-DOF systems.

Vu Dinh Huong et al. [17] employed the general bandwidth method to develop a precise formula for predicting the viscous damping ratio based on the displacement spectrum of a single-DOF system. This formula enhances the determination of the damping ratio by using a power factor instead of the factor of 2 used in the HPB method. Analysis of the single-DOF system showed that the method provides a more accurate damping estimate than the HPB method and is effective for estimating high levels of damping. Subsequently, the authors applied the general bandwidth method to identify hysteresis damping from acceleration frequency response functions (FRF) [18, 20]. Later, Wu [19] also analyzed the displacement spectrum and proposed a formula for estimating the damping ratio based on the power ratio, similar to the formula in [17]. The authors then examined how the power ratio influences the accuracy of damping estimation formulas in multi-DOF systems.

The EFB method is a recently developed approach for estimating damping. It relies on the real part of the frequency response function (FRF) to create formulas for estimating damping parameters. This paper introduces an approximate formula for estimating the viscous damping ratio using the EFB method. Numerical experiments were conducted on single-degree-of-freedom (DOF) and multi-DOF structures, and the results of viscous damping estimation using the proposed formula are compared with existing formulas of the half-power method.

2. Half-power bandwidth method

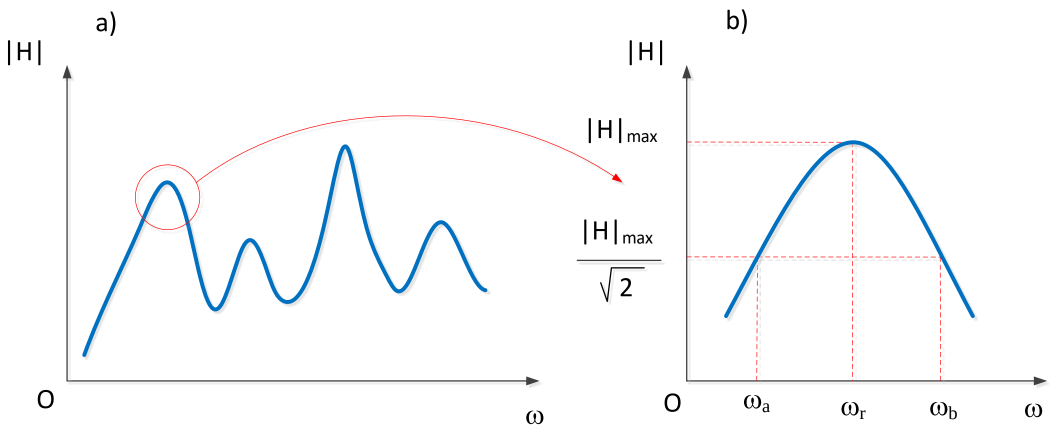

Figure 1. *Half-power bandwidth method*

The classical formula for damping estimation based on the displacement FRF of single-DOF system has the following form [1-3]:

$$\xi = \frac{\omega_b - \omega_a}{2\omega_r} = \frac{b}{2} \qquad (1)$$

where b is the frequency bandwidth; ω_a and ω_b are two frequencies at the amplitudes equal to $1/\sqrt{2}$ the maximum amplitude (at resonance frequency ω_r).

By continuously expanding the square root expressions, in 2011, Wang [6] proposed a third-order equation to determine the viscous damping ratio from the displacement FRF as follows:

$$4\xi^3 + 2\xi = b \qquad (2)$$

The formula for determining ξ according to (2) has significantly reduced the error, but the condition for applying (2) has been shown by Papagiannopoulos [2] and Wu [5] to be $\xi < 0.383$.

Furthermore, the error when estimating the larger damping ratio ($\xi \geq 0.35$) is still greater than 10%.

3. Extreme frequency bandwidth method

Dynamic equilibrium equation of a single-DOF system with viscous damping:

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=p(t) \quad (3)$$

Transform to the frequency domain, substituting equations u(t) = U(ω).e$^{i\omega t}$ and p(t) = P(ω).e$^{i\omega t}$ into equation (3), we obtain:

$$(-m\omega^2+ic\omega+k).U(\omega).e^{i\omega t}=P(\omega).e^{i\omega t} \quad (4)$$

The ratio between displacement and applied force in the frequency domain:

$$H(\omega)=\frac{U(\omega)}{P(\omega)}=\frac{1}{k-m\omega^2+ic\omega} \quad (5)$$

H(ω) is called the displacement (or receptance) frequency response function (FRF) of the single-DOF system. FRF is a complex quantity that depends on frequency and it can also be expressed as.

$$H(\omega)=\frac{1/k}{1-\frac{\omega^2}{\omega_0^2}+i2\xi\frac{\omega}{\omega_0}}=\frac{1}{k}\frac{1-\eta^2}{(1-\eta^2)^2+(2\xi\eta)^2}-i\frac{1}{k}\frac{2\xi\eta}{(1-\eta^2)^2+(2\xi\eta)^2} \quad (6)$$

with, $\eta=\omega/\omega_0$

Real part of the receptance FRF is:

$$\mathrm{Re(H)}=\frac{1}{k}\frac{1-\eta^2}{(1-\eta^2)^2+(2\xi\eta)^2} \quad (7)$$

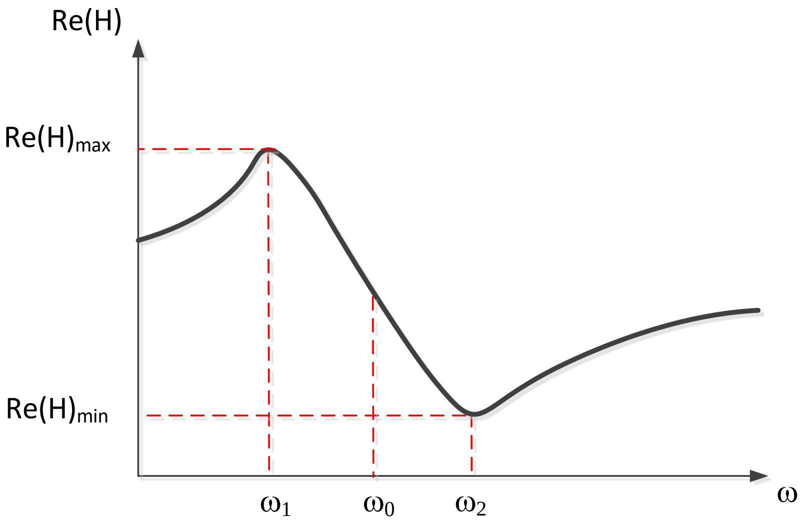

Figure 2. *Extreme frequency bandwidth method*

The graph of the Re(H) function is shown in Figure 2, where ω_1 and ω_2 are the extreme frequencies. The function Re(H) reaches an extreme value when its derivative is zero:

$$\frac{\mathrm{d(Re(H))}}{d\eta}=0\Rightarrow\eta^4-2\eta^2+1-4\xi^2=0 \quad (8)$$

The solution of the equation (8) will then give the following two roots:

$$\eta_{1,2}=\sqrt{1\mp2\xi}\ \text{(with } \xi\leq0.5) \quad (9)$$

Using Taylor expansion for the expressions (9), we have:

$$\frac{\omega_1}{\omega_0}=\eta_1=\sqrt{1-2\xi}=1-\frac{1}{2}(2\xi)-\frac{1}{2}\xi^2-\frac{1}{2}\xi^3-\frac{5}{8}\xi^4+\cdots \quad (10)$$

$$\frac{\omega_2}{\omega_0}=\eta_2=\sqrt{1+2\xi}=1+\frac{1}{2}(2\xi)-\frac{1}{2}\xi^2+\frac{1}{2}\xi^3-\frac{5}{8}\xi^4+\cdots \quad (11)$$

From (10) and (11), obtain the formula to determine the natural frequency of the structure according to the extreme frequencies:

$$\frac{\omega_2+\omega_1}{\omega_0}=\eta_2+\eta_1=2-\xi^2+\frac{5}{4}\xi^4+... \quad (12)$$

When the damping is small (ξ << 1), ignoring the 2nd-order infinitesimal quantity of ξ and its higher-order terms in (12), we get the natural frequency approximately equal to the average of the extreme frequencies:

$$\omega_0\approx\frac{\omega_2+\omega_1}{2} \quad (13)$$

Combining (10) and (11) yields the extreme frequency bandwidth and ignoring the 5th-order infinitesimal quantity of ξ we get a third-order equation to determine the damping ratio:

$$\beta=\eta_2-\eta_1=2\xi+\xi^3 \quad (14)$$

In which, β is extreme frequency bandwidth of real part of receptance FRF:

$$\beta=\frac{\omega_2-\omega_1}{\omega_0} \quad (15)$$

Finally, the cubic equation (14) has a solution:

$$\xi=\left(\frac{\beta}{2}+\sqrt{\frac{\beta^2}{4}+\frac{8}{27}}\right)^{1/3}-\frac{2}{3}\left(\frac{\beta}{2}+\sqrt{\frac{\beta^2}{4}+\frac{8}{27}}\right)^{-1/3} \quad (16)$$

Equation (16) is an approximate formula, proposed to determine the damping ratio from extreme frequency bandwidth of real part of displacement FRF. The estimated range of the damping ratio of this method according to (9) is ξ ≤ 0.5.

4. Comparison of the accuracy of damping estimation methods in the single-DOF system

Consider a single-DOF system with natural frequency ω_0 = π (rad/s). First, assume the damping ratio ξ is a value from 0.05 to 0.5, then equation (6) gives the dimensionless FRF (the stiffness *k* can be assumed to be 1). From this, the amplitude of the FRF and the corresponding half-power frequencies can be determined, and similarly the real part of the FRF and the corresponding extreme frequencies can be obtained. Finally, the damping ratio can be estimated according to the above formulas.

The results of the damping estimation by the methods are presented in Table 1. In which, the first column is the assumed ξ value, the second and third columns show the ξ value calculated by the classical HPB method using the classical formula (1) and its corresponding error (in %); the fourth and fifth columns are the ξ values calculated by the third-order formula (2) and the error; the sixth and seventh columns show the damping ratio value and the error estimated by the proposed formula from the extreme frequency bandwidth method in this paper.

Table 1. *Estimation of damping ratio in the single-DOF system*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.050	0.5%	0.050	0.01%	0.050	0.12%
0.1	0.102	2.1%	0.100	0.06%	0.101	0.51%
0.15	0.157	4.8%	0.150	0.29%	0.152	1.19%
0.2	0.218	9.1%	0.202	0.91%	0.204	2.22%
0.25	0.289	15.6%	0.256	2.23%	0.259	3.69%
0.3	0.377	25.6%	0.315	4.84%	0.317	5.79%
0.35	0.502	43.5%	0.387	10.46%	0.381	8.79%
0.383	0.707	84.7%	0.483	26.00%	0.427	11.56%
0.4	-	-	-	-	0.453	13.35%
0.45	-	-	-	-	0.546	21.24%
0.5	-	-	-	-	0.771	54.18%

Figure 3 shows more clearly the error of the damping estimation methods when the damping ratio changes. Accordingly, the classical formula (1) gives the largest error and is only suitable for structures with a damping ratio less than 0.15. The formula (2) proposed by Wang [4] can estimate well for a damping ratio up to 0.3. However, when the damping ratio is larger than the critical value $\xi_{th} \approx 0.383$, the both formulas give large errors and are unstable. The proposed formula estimates the damping quite accurately, similar to Wang's third-order formula when the structure has damping level less than 0.3. The proposed method can estimate the damping ratio up to 0.5, while HPB method can only estimate the damping ratio up to 0.383.

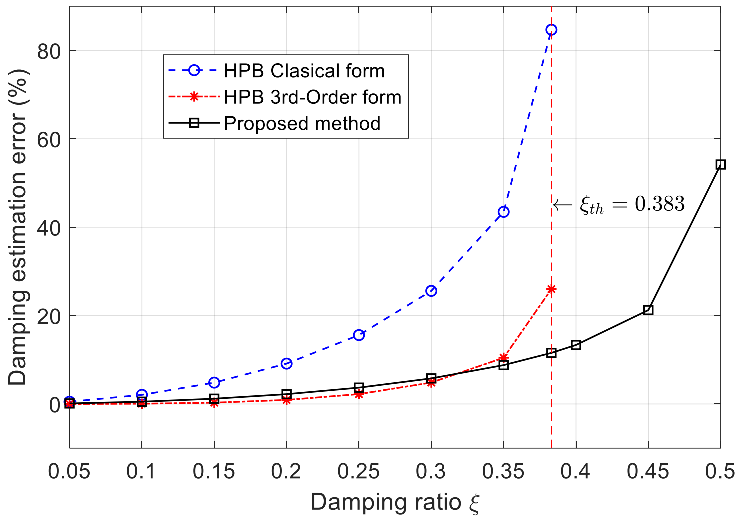

Figure 3. *The error of the damping estimation methods*

5. Numerical simulation on structures

5.1 Simulated vibration measurement data

Consider the structure of a 3-story reinforced concrete building as shown in Figure 4, which is subjected to dynamic load. The structure has the following parameters: beam cross-section size bxh = 0.22 x 0.5 cm, column cross-section 0.22 x 0.22 cm, floor height is 5 m, span width is 6 m. Concrete material has modulus of elasticity E = 2.5e7 (kN/m^2), specific weight ρ = 25 (kN/m^3).

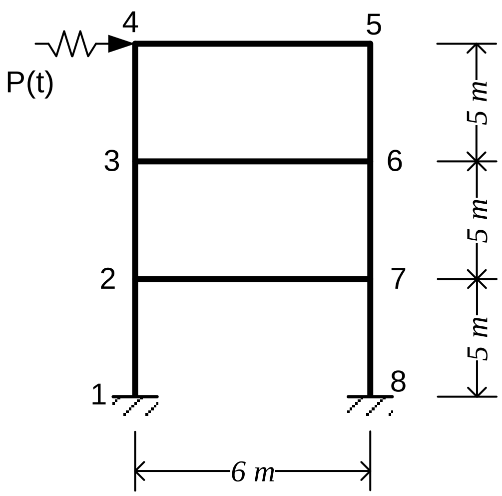

Figure 4. *The 3-story building*

The system is subjected to a half cycle of a sinusoidal load with period 2.0 s as shown in Figure 5a. Assume that the proportional damping model has the same damping levels for all modes. The displacement simulation measurement data at the nodes is obtained by time history analysis of the finite element model using the direct integration method. The horizontal displacement at node 5 with small damping level (ξ = 0.05÷0.15) is shown in figure 5b and with large damping level (ξ = 0.2÷0.4) it is shown in Figure 6b.

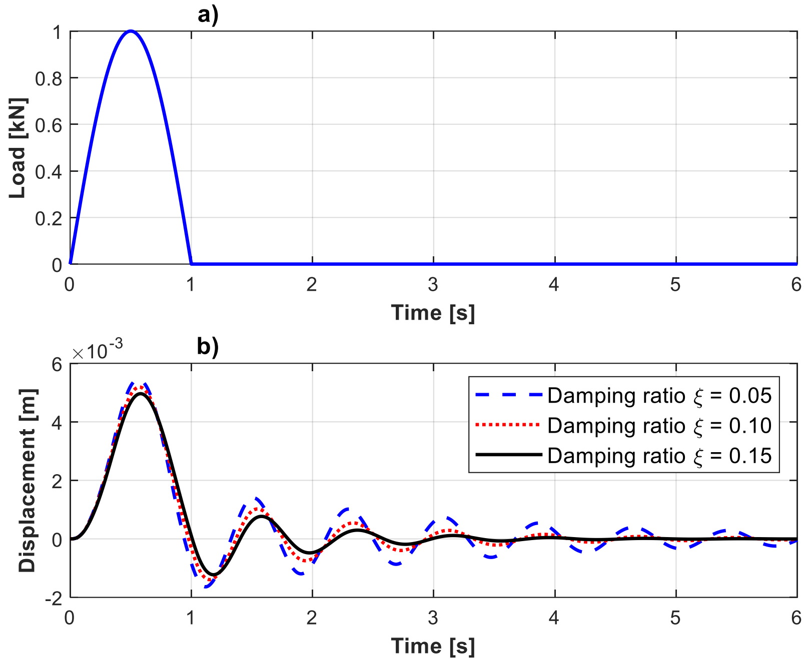

Figure 5. *The load and displacement at node 5 with small damping level*

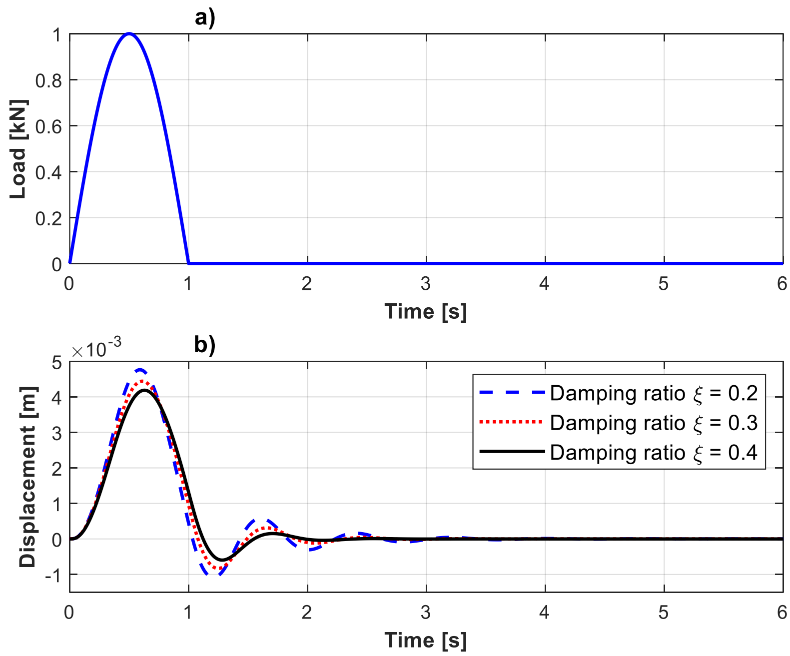

Figure 6. *The load and displacement at node 5 with large damping level*

5.2 The result of damping estimation

Simulate the displacement frequency response function using Fourier transform of the input-output responses as follows:

$$H(\omega) = \frac{U(\omega)}{P(\omega)} \quad (17)$$

where U(ω) and P(ω) are the Fourier transforms of the displacement u(t) and the dynamic load p(t) respectively.

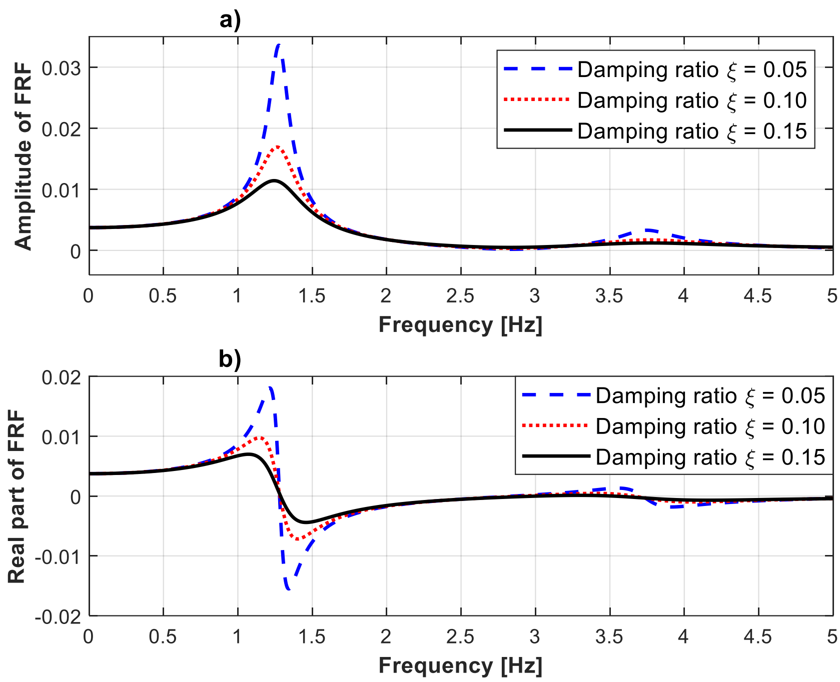

Figure 7. *The FRF with small damping*

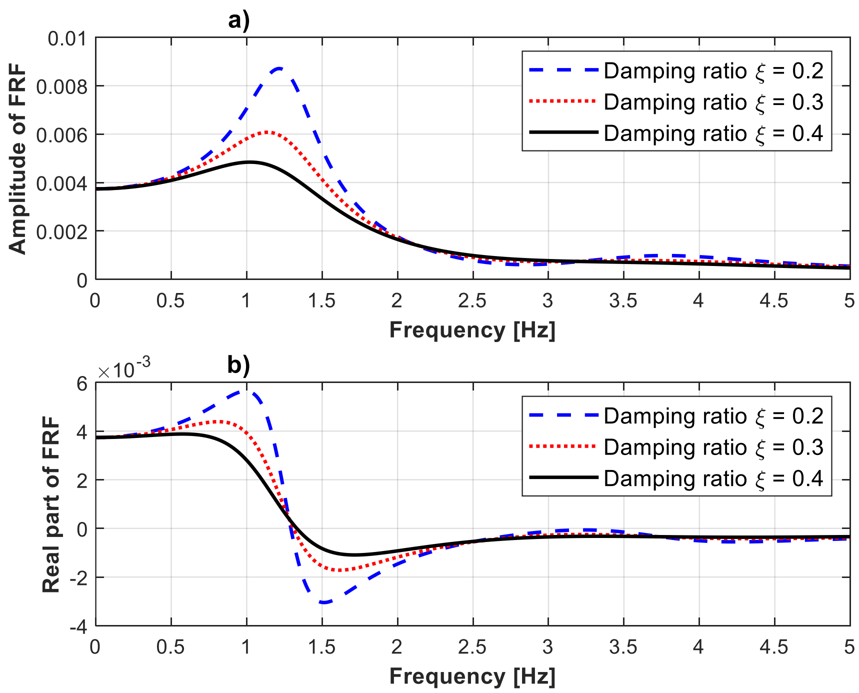

Figure 8. *The FRF with large damping*

The graphs of amplitude and real part of the FRF for horizontal displacement at node 5 are shown in Figure 7 in case the assumed damping level is small. The corresponding graphs in case of large damping level are shown in Figure 8.

From the graphs, the half-power frequencies in amplitude of the FRF and the extreme frequencies in real part of FRF can be determined. From there, the damping ratios can be estimated using the half-power bandwidth method and extreme frequency bandwidth method. Finally, calculate the damping ratios according to the above formulas and evaluate its error. The damping estimation results for the first mode are provided in Table 2, and those for the second mode are in Table 3.

Table 2. *Damping estimation error for the first mode*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.0502	0.4%	0.0499	0.1%	0.0500	0.03%
0.1	0.1025	2.5%	0.1005	0.5%	0.1000	0.05%
0.15	0.1596	6.4%	0.1525	1.7%	0.1513	0.86%
0.2	0.225	12.5%	0.207	3.6%	0.203	1.7%
0.3	0.409	36.3%	0.334	11.4%	0.316	5.2%
0.4	-	-	-	-	0.452	13.0%

Table 2 show that the damping estimation error for the first mode using the proposed method is significantly smaller compared to the classical and 3-rd order formulas based on HPB method. In addition, the proposed method provides a larger damping estimation range than the existing formulations. For the first mode, the classical formula gives good results when estimating small damping levels (about less than 0.1), the 3rd-order formula can estimate larger damping levels (about 0.2), and the proposed formula can estimate damping ratios up to 0.3 with an error of about 5%. Additionally, at a high damping level (0.4), the proposed method has an error of 13%, while the HPB method does not provide a solution.

Table 3. *Damping estimation error of the second mode*

ξ assumed	ξ from Eq. (1) and error %		ξ from Eq. (2) and error %		ξ from Proposed Eq. (16) and error %	
0.05	0.049	1.9%	0.049	2.4%	0.049	2.1%
0.1	0.096	3.8%	0.094	5.5%	0.095	5.3%
0.15	0.141	6.0%	0.136	9.3%	0.135	9.8%
0.2	0.183	8.6%	0.172	13.8%	0.171	14.7%
0.3	0.338	15.4%	0.248	20.8%	0.234	22.1%
0.4	-	-	-	-	0.284	29.1%

Table 3 shows that all three formulas give good results only when estimating small damping levels (less than 0.1). As the damping level increases, the errors of the formulas also increase. The proposed formula has lower accuracy than the 3rd-order formula, but the difference is quite small. On the other hand, at high damping levels, the classical formula for damping estimation performs better for the second mode compared to the other two formulas, although the error is still relatively large (from 6% to 15.4%). The proposed method can still estimate a damping level of 0.4, but with an error as high as 29.1%.

Tables 2 and 3 also show that the proposed method is highly effective for estimating damping in the first mode, even at high damping levels. However, the method performs less effectively when applied to the second mode, especially at high damping levels.

5. Conclusion

The paper proposes a formula for estimating the viscous damping ratio of a structure from the extreme frequency bandwidth of the real part graph of the displacement FRF. For a single-DOF system, the proposed formula gives more accurate damping estimation results than the classical formula, and the damping estimation range is extended to a damping level of 0.5. In a multi-DOF system, the proposed formula can estimate the damping ratio for the first mode with small errors, even with a large damping level. However, the damping ratio estimation results for the second mode increase significantly as the damping level increases.

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