ENHANCED METHOD FOR OPERATIONAL MODAL IDENTIFICATION BASED ON BLIND SOURCE SEPARATION

NÂNG CAO HIỆU QUẢ NHẬN DẠNG CÁC THAM SỐ DAO ĐỘNG DỰA TRÊN KỸ THUẬT TÁCH NGUỒN MÙ

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Abstract: *Blind source separation (BSS) based identification methods draw increasing attention as an alternative tool for operational modal analysis. However, their implementation is rare in literature, especially in cases where the number of sensors is limited (the underdetermined problem) or harmonic excitations are involved. This paper proposes an enhanced modal identification procedure based on the BSS technique to handle even the underdetermined problem or the presence of harmonic excitation. Numerical simulation and experimental example show that whether in determined situations or the presence of harmonic excitation, the proposed method performs accurate and robust identification.*

Keyword: *modal analysis, underdetermined problem, harmonic excitation.*

Tóm tắt: *Các phương pháp nhận dạng dựa trên kỹ thuật tách nguồn mù (BSS) ngày càng thu hút nhiều chú ý như một công cụ thay thế cho các kỹ thuật nhận dạng dao động. Tuy nhiên, việc triển khai chúng còn chưa phổ biến trong các ứng dụng nhận dạng, đặc biệt là trong những trường hợp số lượng cảm biến bị hạn chế (vấn đề chưa xác định) hoặc khi có mặt của kích động điều hòa. Bài báo đề xuất một quy trình nhận dạng dao đông dựa trên kỹ thuật BSS để xử lý các vấn đề chưa xác định hoặc dưới sự hiện diện của kích động điều hòa. Mô phỏng số và ví dụ thực nghiệm cho thấy rằng trong các tình huống chưa xác định hoặc sự hiện diện của kích động điều hòa, phương pháp được đề xuất thực hiện nhận dạng chính xác và hiệu quả.*

Từ khóa: *phân tích dao động, vấn đề chưa xác định, kích động điều hòa.*

1. Introduction

One of the main problems in structural dynamics is to determine the modal parameters, such as frequency, damping, and modal shape. Among many modal analysis methods [1], output-only methods [2,3] are required when only the structural responses are available in many situations, when input measurements are extremely difficult or even impossible to measure, or when it is impractical to apply controllable excitations for modal identification of large-scale structures.

Blind source separation techniques emerged in the 1990s in the audio field, used to extract individual sound sources from records. This technique has become increasingly popular in modal identification due to their simplicity, efficiency, non-parametric nature, and no prior information required about the dynamic system. Two early BSS techniques, namely independent component analysis (ICA) [4] and second-order blind identification (SOBI) [5], are successfully applied to perform output-only modal identification. However, these methods require the number of measurements to equal or exceed the number of operating modes, which may not be suitable for large-scale structures with a limited number of measurement sensors.

In addition, a fundamental assumption of operational modal analysis methods is that external excitation is white noise. This assumption implies that the excitation does not drive the system at any specific frequency and therefore any identified active frequency reflects the modal response of the structure. However, in practice, some harmonic disturbances, such as an adjacent machine operating at a particular frequency, may drive the structure at that frequency. Therefore, these components need to be detected and removed during the modal identification procedure.

In this study, an enhanced BSS-based procedure is proposed for OMA suitable for practical applications. This procedure ensures robust estimation of modal parameters, detection and removal of harmonic excitations. With these main objectives, this paper is organized in the following structure. Section 2 introduces the basic theorem of BSS, harmonic detection technique and proposed procedure. Section 3 introduces a novel BSS method. Numerical example and experimental validation are carried out in Section 3 and section 4, respectively. Then, the conclusion is followed in Section 5.

2. Estimation of modal parameters based on BSS

2.1. Bind source separation (BSS)

BSS is a powerful tool for separating mixed signals when the source and mixing process are unknown. The simple form of BSS in the absence of noise is to determine a mixing matrix and recover the component sources from their linear mixtures which can be expressed as follows:

$$\mathbf{X}(t) = \mathbf{AS}(t) \tag{1}$$

The aim of BBS is to extract both the n_s original sources *s(t)* and the mixing matrix *A* from the n_x measured mixtures. Depending on the relation between the number of measurements n_x and the number of sources ns, BSS problems can be classified as overdetermined case when $n_x > n_s$, determined case when $n_x = n_s$, or underdetermined case when $n_x < n_s$.

Eq. (1) is similar to the classical modal superposition technique; the vibration measurement **X** can be decomposed through the mode shape matrix $\mathbf{\Phi}$ into single mode response signals q(t):

$$\mathbf{X}(t) = \mathbf{\Phi q}(t) = \sum_{i=1}^{n} \boldsymbol{\varphi}_i \mathbf{q}_i(t) \tag{2}$$

In order to represent the sparsity of the measured signals, the Short Time Fourier Transform (STFT) was applied to Eq. (1).

$$\mathbf{X}(t, f) = \mathbf{AS}(t, f) \tag{3}$$

In the case of two mixtures consisting of three source components, the above equation can be expressed as:

$$\begin{Bmatrix} \mathbf{X}_1(t, f) \\ \mathbf{X}_2(t, f) \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{Bmatrix} S_1(t, f) \\ S_1(t, f) \\ S_1(t, f) \end{Bmatrix} \tag{4}$$

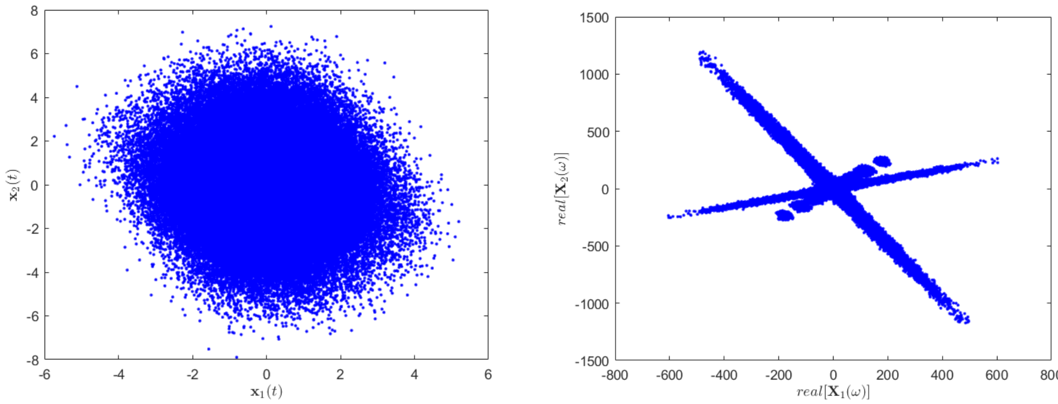

Figure 1. *Scatter plot of two mixtures in the time domain (left) and in the time-frequency domain (right)*

The original form of the measured mixtures generally does not fit the assumption about sparsity in the time domain, but they exhibit the sparsity in the transformed domain as shown in Figure 1 (each straight line represents a latent component). The BSS method is achieved by two main stages: mixing matrix estimation and source recovery [7]. The first step is to estimate the mixing matrix A using clustering techniques [8-10] (hierarchical clustering, K-means algorithm, Fuzzy C-Means clustering…) from the scatter plot of the TF coefficients, and then the source can be recovered using ℓ1-norm minimization in the Time-Frequency domain (Eq. 5).

$$\mathbf{S}^*(t, f) = \operatorname{argmin} \|\mathbf{S}(t, f)\|_{l_1} \text{ subject to } \mathbf{A}^*\mathbf{S}(t, f) = \mathbf{X}(t, f) \tag{5}$$

Finally, source signals (modal responses) can be reconstructed to the time domain by the inverse short-time Fourier transform (iSTFT). The modal parameters can be identified from recovered signals by using either single-mode curve fitting in the frequency domain or logarithmic decrement method in the time domain.

2.2 Kurtosis

The conventional kurtosis value can be used to distinguish modal responses and harmonic components [11-13]. Kurtosis is a measure of the tailedness of the probability distribution of a real-valued random variable. The kurtosis is defined as the fourth central moment of the stochastic variable as follows [14]:

$$k = \frac{E\left\{x^4\right\}}{\left(E\left\{x^2\right\}\right)^2} \tag{6}$$

where *E* is the expectation operator

The Probability Density Function (PDF) of the response of a structural mode will be normally distributed (left part of Fig. 2), and the kurtosis k = 3. The PDF p is given as follows:

$$p\left(x \mid \mu, \delta\right) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\delta^2}} \tag{7}$$

where μ and δ are the mean and standard deviation of x, respectively.

The PDF will have two distinct peaks in the case of a pure harmonic component (right part of Fig. 2), and the kurtosis k = 1.5. The PDF p is given as follows:

$$p(x,a) = \begin{cases} 0 & |x| > a \\ \dfrac{1}{\pi \cos\left(\arcsin\left(\dfrac{x}{a}\right)\right)} & |x| \le a \end{cases} \tag{8}$$

where *a* is the amplitude of the harmonic component.

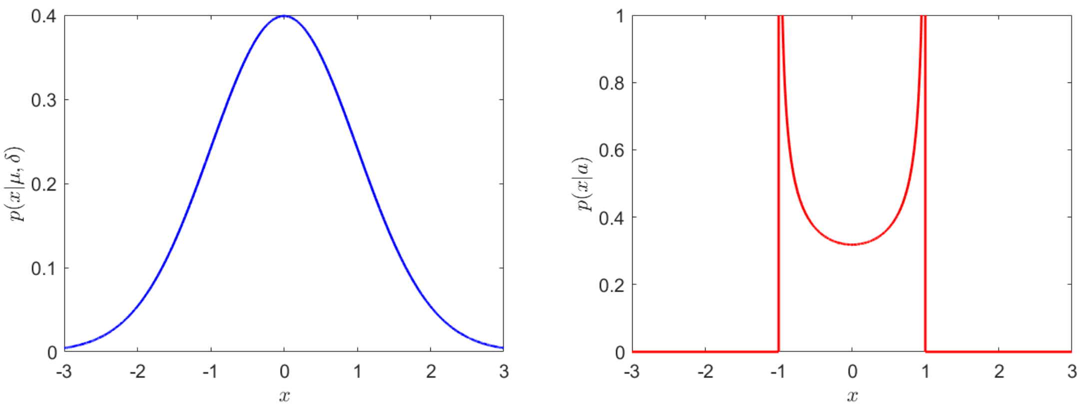

Figure 2. *Normalized PDF of the response of a structural mode (left) and a harmonic component (right)*

2.3 Proposed procedure

The steps in the proposed procedure for determining the modal parameters and harmonic components are given as follows:

- Measure structural responses;

- Perform short-time Fourier transform of the measured signal into the time-frequency domain;

- Apply BSS technique to separate components (obtain mode shape and mode response in time-frequency domain), and then recover mode response to time domain using iSTFT;

- Distinguish between the harmonic component and the structural component by the kurtosis value;

- Remove the harmonic components (with kurtosis value = 3).

3. Numerical test

To illustrate the effectiveness of the proposed procedure, an example of a 2-DOF mass–spring–damper system with the mass matrix M, the stiffness matrix K, and damping matrix C is considered:

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix};\ K = \begin{bmatrix} 800 & -800 \\ -800 & 2400 \end{bmatrix}; C = \begin{bmatrix} 0.44 & -0.04 \\ -0.04 & 0.52 \end{bmatrix}$$

The two exact natural frequencies and the three damping ratios are presented in Table 1. The system is subjected to a random excitation accompanied by a harmonic excitation. Structural responses are simulated for a duration of 60 s with a sampling rate of 200 Hz. The responses are presented in Figure 3.

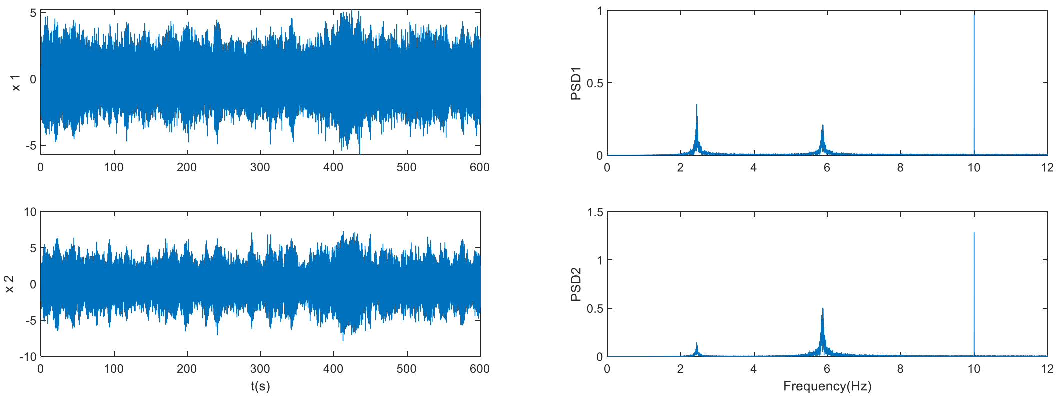

Figure 3. *Simulated signals in the time domain (left) and the frequency domain of 2-dof system*

The Modal Assurance Criterion (MAC) is used to determine the errors of the estimated mode shapes. The MAC is defined as follows [15]:

$$MAC_i = \frac{\left(\mathbf{\Phi}_i^T \bar{\mathbf{\Phi}}_i\right)^2}{\mathbf{\Phi}_i^T \mathbf{\Phi}_i \bar{\mathbf{\Phi}}_i^T \bar{\mathbf{\Phi}}_i} \quad (9)$$

where $\mathbf{\Phi}_i$ and $\bar{\mathbf{\Phi}}_i$ are the i*th* reference mode shape and estimated value, respectively.

The developed technique is performed to identify modal parameters. The individual components are shown in Figure 4. The identified modal parameters with their exact values are shown in Table 1. This indicates that the proposed method is effective for the underdetermined case (two signals vs. three latent components) as well as the presence of harmonic excitation.

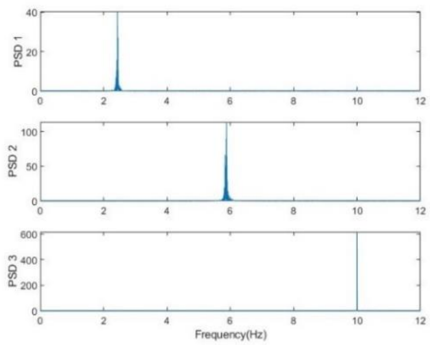

Figure 4. *Separated components in the frequency domain*

4. Experimental test

In this section, the authors employ an experiment to validate the effectiveness of the proposed method. The experiment was conducted at the Laboratory of Mechanics and Energy of University Paris-Saclay. The physical parameters of the beam are the length 1005 mm, the width 42mm, the height 10mm, Young's modulus 200 GPa, and mass density 7850 kg/m^3.

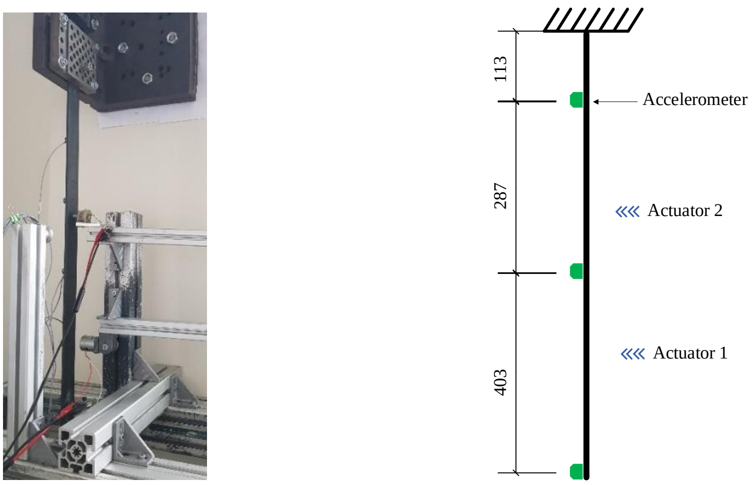

Figure 5. *The experimental setup*

The data is the acceleration of the beam used for the operational modal analysis. The authors conducted the experimental test as shown in Figure 5. The cantilever beam was simultaneously subjected to a random excitation (actuator 1) and a harmonic loading at 20 Hz (actuator 2). The structural responses were collected using three B&K Type 4533-B-001accelerometers mounted along the beam at a sampling rate of 2048 Hz. The measurements are shown in Fig. 6. The proposed procedure is then applied to the measured data. Scatter plot of two signals in the time domain (left) and in the time-frequency domain (right) of Fig. 7. The original form of signals does not fit the sparsity assumption, but it shows approximately directions of four straight lines in the transformed (three structural modes and a harmonic component). The separated components are shown in Fig. 8. Modal parameters are well identified by the proposed procedure compared to the reference results [13] (Table 1).

The analysis results show that the enhanced method is capable of analyzing vibration signals, accurately determining mode parameters for underdetermined problems and the presence of harmonic excitation.

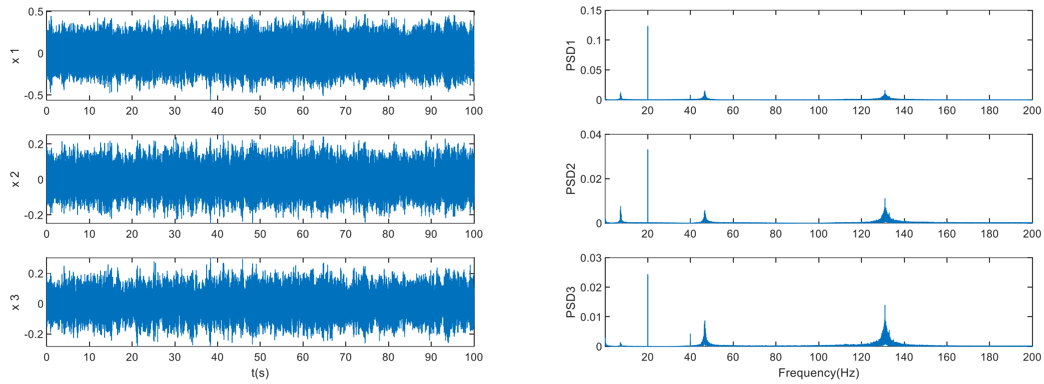

Figure 6. *Measured signals in the time domain (left) and the frequency domain (right)*

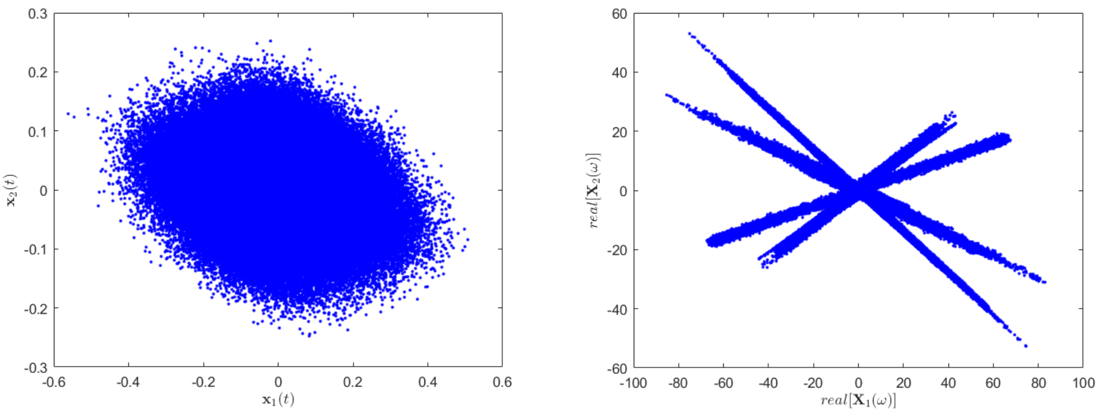

Figure 7. *Scatter plot of two signals in the time domain (left) and in the time-frequency domain (right)*

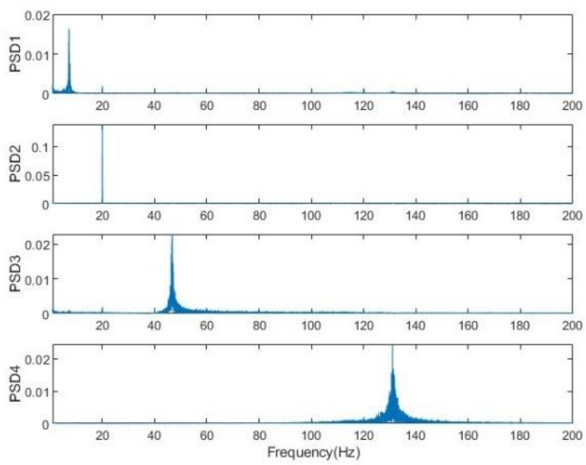

Figure 8. *Separated components in the frequency domain*

Table 1. *Modal parameters estimated by the proposed method*

Test	Mode	Reference values		Identified values		MAC	Kurtosis	Conclusion
		f(Hz)	*ξ (%)*	*f(Hz)*	*ξ (%)*			
Numerical test	1	2.436	0.69	2.43	0.69	1,00	2.95	Structural
	2	5.88	0.36	5.88	0.36	1,00	2.98	Structural
	3	10,00	-	10,00	-	-	1.50	Harmonic
Experimental test	1	7.28	1.11	7.28	1.10	0.98	3.01	Structural
	2	20,00	-	19.99	-	-	1.56	Harmonic
	3	46.54	0.32	46.51	0.29	0.99	3.14	Structural
	4	130.60	0.52	130.67	0.53	0.99	3.08	Structural

5. Conclusions

This study proposes a BSS based method to perform output-only modal identification. The method presumes that sources are sparsely represented in the transformed domain. The sparse representation of the measured signals in the transformed domain allows the estimation of the mode shape matrix using a clustering algorithm. The modal responses can then be estimated by minimizing the ℓ1-norm minimization.

Compared with the existing BSS-based modal analysis method, the proposed method is not only suitable for the determined cases but also capable of identifying the modal parameters when the sensor is limited compared to the number of active modes, even in the presence of harmonic excitations.

Both numerical and experimental studies confirm the effectiveness of the proposed method in identifying structural modes. It is also shown to be robust to random excitation and harmonic excitation.

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